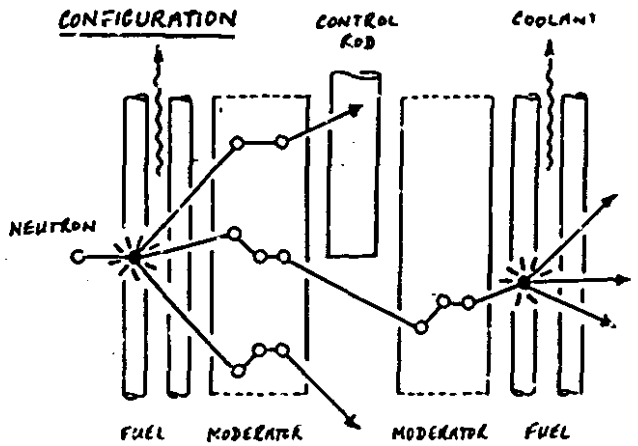
REFERENCE DATA

REACTOR PRINCIPLES

PRINCIPLES

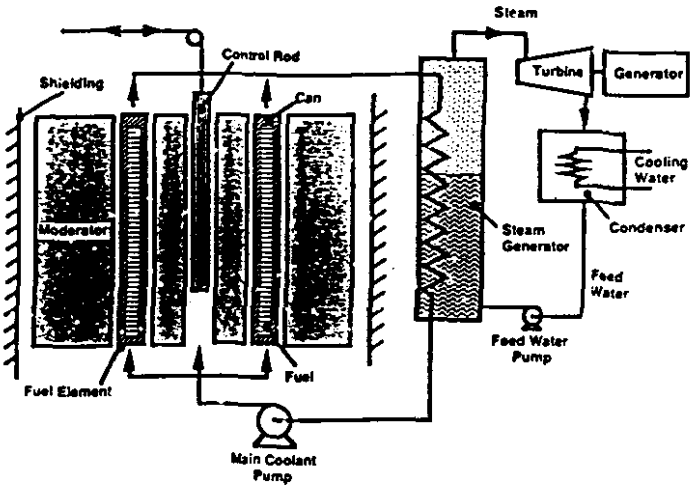
- NEUTRON CHAIN REACTION
- CONTROL OF NEUTRONS
- MODERATION OF NEUTRONS
- REMOVAL OF ENERGY



CLASSIFICATION OF POWER REACTORS

	MODERATOR	COOLANT	FUEL: Natural U	Enriched U	Thorium-U	Pu/U
FAST REACTORS	None	Na				LMFBR
		He				GCFR
THERMAL REACTORS	Heavy Water	CO_2		KKN, EL4		
		HB-40	CANDU-OCR			
		D_2O	CANDU	Atucha		
		H_2O	CANDU-BLW	SGHW		ATR
	Light Water	H_2O (Boiling)		BWR		
		H_2O (Pressurised)		PWR	LWBR	
	Graphite	He		HTGR	THTR	
		H_2O		RBMK		
		CO_2	Magnox	AGR		
	Salt	Molten Salt			MSBR	

BASIC REACTOR COMPONENTS



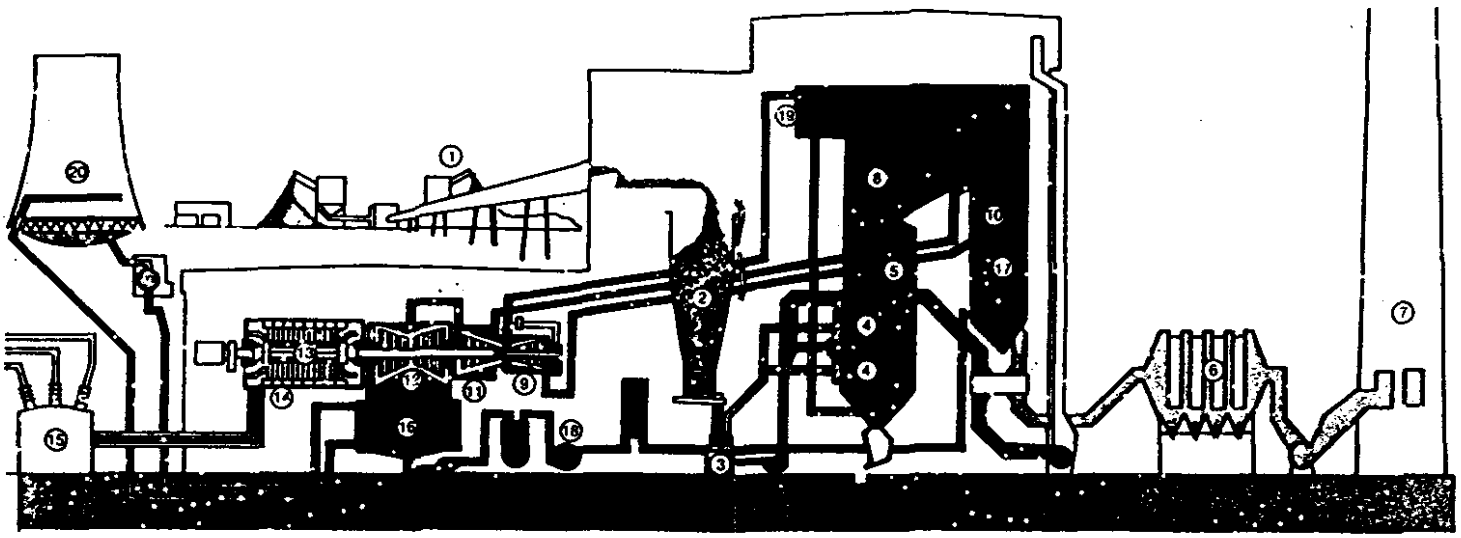
REACTOR TYPES

LWR	LIGHT WATER REACTORS
BWR	Boiling Water Reactor
PWR	Pressurized Water Reactor
HWR	HEAVY WATER REACTORS
CANDU	Canadian Deuterium Uranium Reactor
CANDU-PHW	CANDU - Pressurized Heavy Water
CANDU-BLW	CANDU - Boiling Light Water
SGHWR	Steam Generating Heavy Water Reactor
GCR	GAS COOLED REACTORS
AGR	Advanced Gas Cooled Reactor
HTR	High Temperature (Gas Cooled) Reactor
FBR	FAST BREEDER REACTOR
LMFBR	Liquid Metal Fast Breeder Reactor
GCFBR	Gas Cooled Fast Breeder Reactor

TYPICAL REACTOR DESIGN CONDITIONS

Plant Type	Output (MWt)	Output (MWe)	Power Density (MW/m³)	Core Dia. (m)	Core Height (m)	Volume (m³)	Normalized Volume (m³)
Magnox	1880	590	0.86	17.4	9.2	2190	4455
AGR	1890	625	3.4	9.3	8.2	557	1069
RBMK	3200	1000	4.2*	11.8	7.0	766	919
THTR	738	300	5.0	5.6	6.0	148	592
HTGR	837	330	6.3	6.0	4.7	133	484
PHWR	2060	625	11.7*	6.3	5.9	183	351
BWR	2570	907	50.0	4.2	3.7	51.3	68
PWR	3500	1140	100.0	3.4	4.0	36.3	38
FBR	3200	1200	500	3.0	0.92	6.5	6.5

* This is the power density averaged over the whole core volume. For comparison with other concepts, this value makes sense only as a scale for the ultimate heat sink capability. A better measure for the power density of a PHWR or RBMK core, from the safety point of view, is that relative to the volume of the pressurized portion of the core. These values are 107 MW/m3 and 44.1 MW/m respectively.



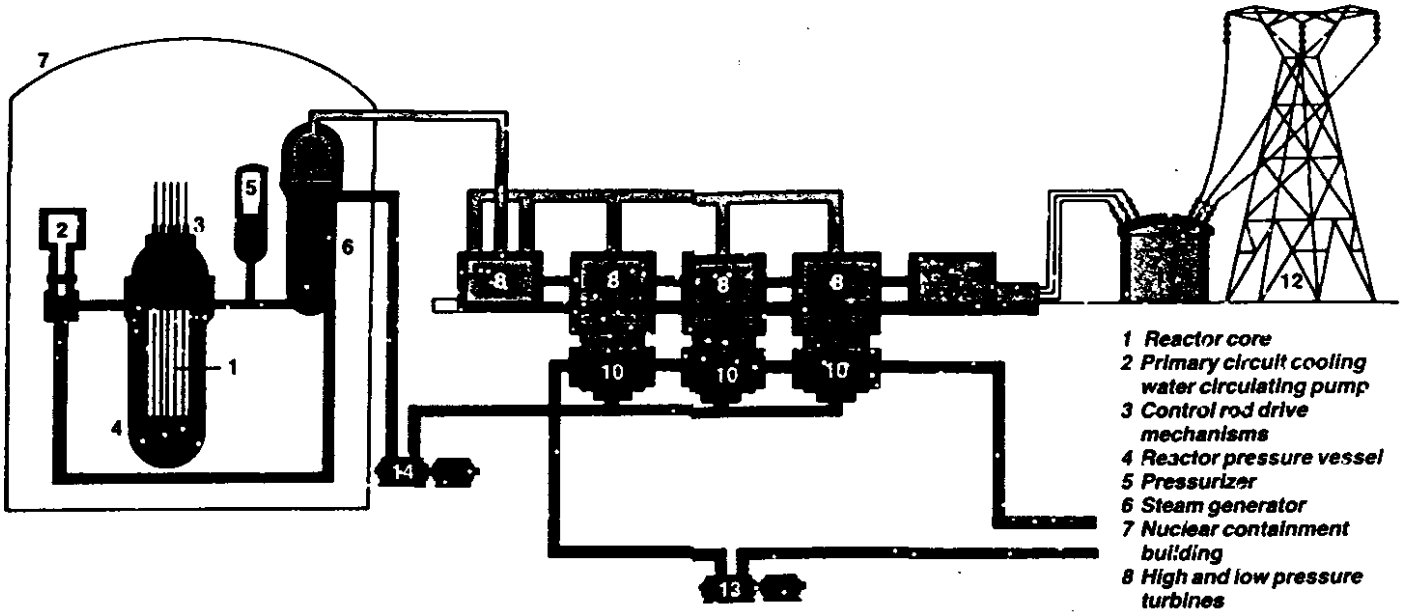
The coal-fired thermal power station: basic cycle

The Boiler
Coal is fed from coal staithes (1) to the boiler bunkers (2) by a conveyor belt, from where it is fed into pulverising mills (3) which grind the coal to powder. This pulverised coal is carried by a stream of air from the mills to the boiler burners (4), where it is blown into the furnace (5) to burn like a gas. The products of this combustion are dust and ash in a ratio of 5:1. The ash falls to the bottom of the boiler where it is sluiced away for treatment and the dust is carried in the flue gases to the precipitators (6) where most of it is collected electrostatically. The cleaned flue gases pass through the chimney (7) to the atmosphere.

Superheated Steam
Heat released by the burning coal is absorbed by boiler feed-water inside many kilometres of tubing which form the boiler walls. The boiler feed-water is converted to steam at a high temperature and pressure. The steam is superheated in further tubes (8). Superheated steam passes to the high-pressure turbine (9) where it is discharged onto the turbine blades. The energy of the steam striking the turbine blades causes the turbine to rotate at 3 000 r.p.m. After exhausting some of its energy in the high-pressure turbine, the steam is reheated in the boiler reheater (10), then passed through the intermediate-pressure turbine (11) and from there to the low-pressure turbine (12). Coupled to the turbine shaft is the rotor of the generator (13). The generator rotor is a cylindrical electro-magnet which is enclosed within a gas-tight housing. The stator (14) consists of large coils of copper bar in which electricity is produced by the rotation of the magnetic field in the rotor. The electricity produced passes from the stator windings to a transformer (15) where the voltage is raised from about 20 kV to the national transmission voltages (275 kV and 400 kV at Matla). Electricity passes through the high-voltage yards from where it is distributed to consumers via the national transmission network.

Cooling and recirculation
After exhausing its energy in the turbines the steam is condensed in a condenser (16) and pumped back as water through the deaerator to the boiler economiser (17) by means of the boiler feed pump (18) for reheating. Water from the economiser is fed to the steam drum (19), which contains both water and steam. Steam is taken off at the steam drum and conveyed to the superheater (8) for further heating before passing to the turbine. The water is fed to the furnace tube walls via headers at the bottom of the combustion chamber to recommence the cycle.

The condensers contain many kilometres of tubing (about 328 km at Matla) through which cold water from the cooling towers (20) is constantly pumped. Heat which the cooling water extracts from the steam circuit is removed by spraying the water out in the lower levels of the cooling towers; the cooled water is then collected in ponds beneath the towers. An upward draught of air within the towers cools the water. The cooled water is then recirculated to the condensers. Some of the sprayed water inevitably rises with the draught of air, forming the familiar clouds of water vapour at the top of cooling towers.



1 *Reactor core*
2 *Primary circuit cooling water circulating pump*
3 *Control rod drive mechanisms*
4 *Reactor pressure vessel*
5 *Pressurizer*
6 *Steam generator*
7 *Nuclear containment building*
8 *High and low pressure turbines*
9 *Generator*
10 *Condensers*
11 *High voltage building*
12 *400 kV transmission line*
13 *Tertiary (sea water) cooling water circulating pump*
14 *Secondary cooling water circulating pump*

Electricity generation using a pressurized water reactor

Heat is generated in the reactor core by the fissioning of nuclei in the fuel elements and is transferred to the primary coolant. The coolant is light water (purified ordinary water) and to prevent it boiling in the reactor it is pressurized by an auxiliary pressurizing system (15,6 MPa at about 320°C). The light water is also the moderator, i.e., it slows down released neutrons from the fissioning U.235 to sustain the chain reaction in the enriched uranium fuel.
The diagram shows how the coolant water is passed through the reactor (1) where it is heated in contact with the fuel rods. The giant 5,4 MW cooling water pumps (2) circulate the water of the primary circuit through the system. Control rods, which control the fission rate and hence the power output, are moved in and out of the core by the control rod drive mechanisms (3) mounted on top of the reactor pressure vessel (4). The domed pressure vessel head and the drive mechanisms are removed during refuelling. Roughly one-third of the fuel is replaced annually.
During operation, the water pressure in the primary circuit is controlled by means of electric heating elements and a "cold" water spray in the pressurizer (5). After passing through the U-tubes of the steam generator (6) the water circulates back through the pumps to complete the circuit at a lower temperature of roughly 280°C.

The entire primary circuit, or nuclear steam supply system (NSSS), consisting of the reactor, three steam generators, the pressurizer and three main coolant pumps, is housed within the massive concrete containment building (7).
Hot water from the reactor passing through the steam generator is cooled when its heat is transferred to the cooler water of the secondary circuit flowing around the U-tubes. This water, being at a lower pressure, boils to form steam, which is dried in the steam generator and then passed along piping to the turbines (8) situated in the turbine hall outside. As the steam expands within the turbines it spins a shaft coupled to the generator (9) which generates electricity. The electricity thus generated is led into the Escom national grid via the high-voltage building (11)
Once the steam has given up its energy, it is condensed back to water in the condensers (10) and pumped back to the steam generators. At Koeberg, seawater is used to cool the condensers, and is pumped through the system at a rate of almost 40 cubic metres a second for each turbo-generator unit. The temperature of the sea in the immediate vicinity of the outfall will be raised by approximately 10°C. Seawater never comes into contact with the nuclear reactor.

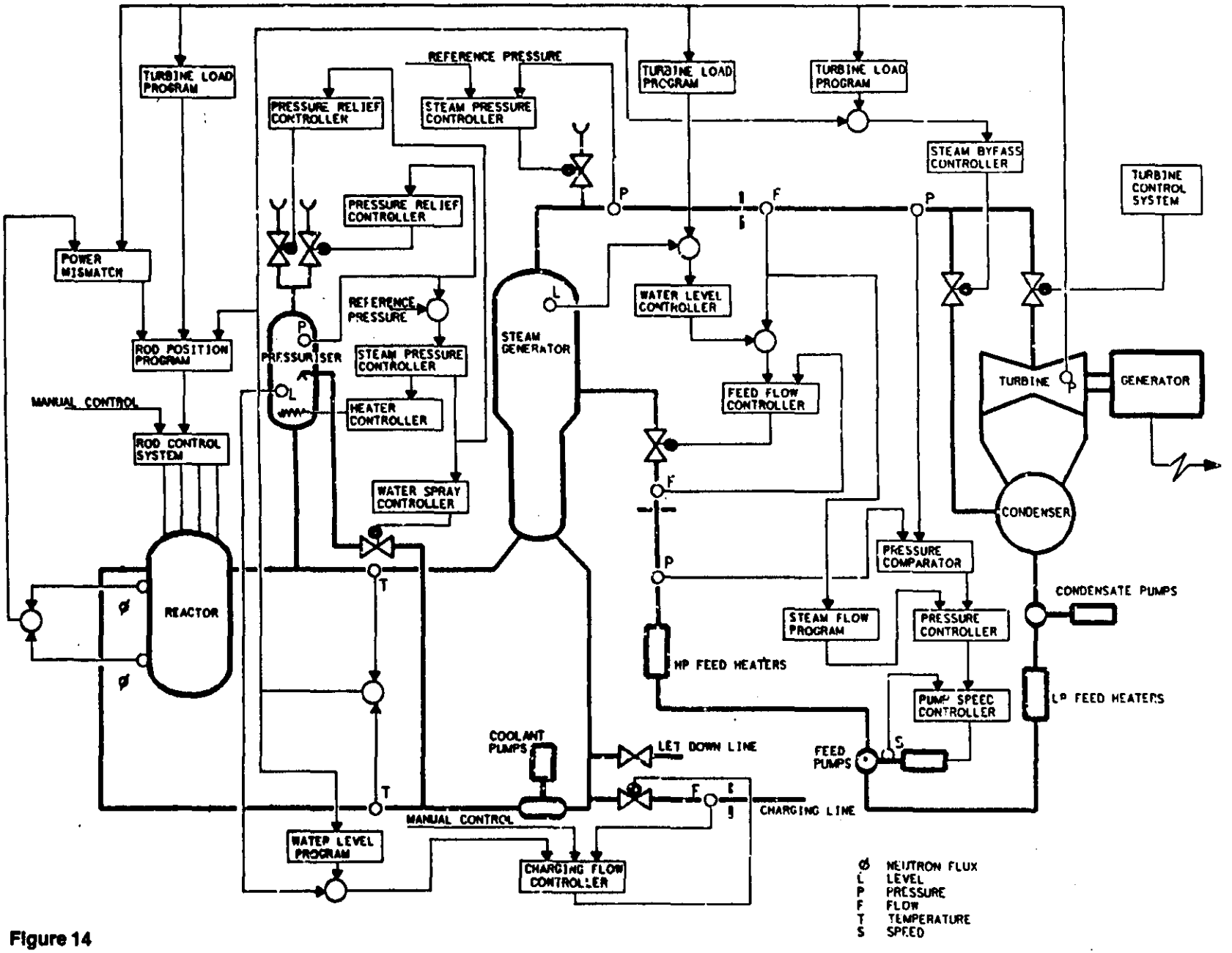


Figure 14
Reactor control system.

KNOW YOUR NUCLEAR REACTORS

1 MAGNOX REACTOR. One of the first types of nuclear reactor to be used for large scale commercial production of electricity was the natural-uranium, gas-graphite system developed in Britain and France. The first commercial reactors of this type are generally known in Britain as the Magnox reactor because the fuel cladding material is a magnesium alloy called Magnox. The reactor core of a 500 MW(e) Magnox nuclear power station would contain around 30,000 fuel elements consisting of natural uranium metal rods canned in Magnox. The fuel elements are stacked in channels in a massive pile of graphite blocks. Carbon dioxide at high pressure is used as a heat transfer fluid (or coolant) in forced circulation through the fuel channels and steam generators. Finning on the fuel cans and steam generator tubing assists heat transfer. Large spherical pressure vessels in steel, connected by ducts to the steam generator units and gas circulators, were used in the early Magnox power stations but in later plants these were superseded by prestressed concrete pressure vessels with an integral arrangement of steam generators (see AGR below). With very large natural uranium reactor cores a more or less continuous fuel changing operation is called for and on-load fuel handling systems were therefore developed. Complex fuel handling machines connect to standpipes which penetrate the top of the pressure vessel and remote grabs on pantograph arms load or withdraw the fuel elements. Neutron absorbing control rods are withdrawn by chain mechanisms from separate channels in the graphite core and can be inserted by gravity for rapid shut down. Due to the large thermal capacity of the graphite core and, in the later stations, the inherent safety of concrete pressure vessels, no special containment building is required to deal with the possible effects of a primary circuit rupture.

2 ADVANCED GAS-COOLED REACTOR. The AGR is a development of the Magnox system designed to raise the maximum allowable temperature of the gas coolant and therefore to improve the steam conditions. Beryllium had been considered as an alternative fuel cladding material to Magnox as it would have allowed higher operating temperatures with natural uranium, but the material proved to be too difficult to work. Stainless steel cladding was therefore adopted and this results in a need to use uranium enriched to give 2 to 3 per cent content of the fissile uranium-235 isotope. The enriched uranium fuel, in the form of sintered oxide pellets, is packed into stainless steel cladding tubes with low profile radial finning. These fuel pins are made up into fuel element assemblies of 36 pins in a graphite sleeve and the fuel element assemblies are loaded into channels in a large graphite moderator stack similar to the Magnox reactors. On-load fuelling is again adopted, mainly in this case to ensure high availability of the plant. The fuelling operation is made simpler by a smaller number of fuel channels and single standpipe access to each channel. The whole of the reactor core and array of once-through steam generators arranged circumferentially around the core, are contained in a prestressed concrete pressure vessel. The impellers of gas circulators are also situated inside the vessel to provide the forced circulation of CO_2 up through the core and down through the steam generators around the core. An alternative configuration with helical steam generators in wall cavities, or "pods," around the main reactor cavity, has been introduced on later AGRs (see HTGR below). The inherent safety of the concrete pressure vessel, the high thermal capacity of the graphite core and the ceramic nature of the oxide fuel make this system suitable for near urban siting without special containment buildings.

3 HIGH TEMPERATURE GAS-COOLED REACTOR. A further evolution of the gas-cooled reactor concepts to yet higher temperatures can be achieved by using helium gas coolant and fuel with a ceramic coating instead of metal cladding. Known variously as the HTGR, HTR or Mk III gas-cooled reactor, the system has been developed internationally by the OECD Dragon Project and in the USA. A variant known as the Thorium High Temperature Reactor (THTR) has been developed separately in Germany using a novel pebble bed core concept. The first commercial stations of this type have been ordered in the USA for operation around 1980. Enriched uranium fuel, in the form of small spheres of uranium carbide, is coated with layers of graphite and silicon carbide to form coated particles which need no further cladding to retain radioactive fission products. The coated particles are packed into axial cavities in hexagonal blocks of graphite which are loaded into the reactor as combined fuel elements and moderator blocks. Axial channels through the fuel-moderator blocks allow the coolant gas to pass down through the core. It is then circulated up through the helical steam generators in the pods of a concrete pressure vessel around the core. US designers envisage off-load fuelling during a shut down period of about two weeks once a year but the British designers are considering on-load fuelling systems similar to that used in the AGR and Magnox stations. Two types of fuel cycle, known as high enrichment and low enrichment schemes, are possible with HTGRs. In the first of these fuel enriched to give 80 or 90 per cent of uranium-235 is mixed with thorium carbide coated particles. During operation the thorium is converted to uranium-233 which is a highly efficient nuclear fuel and can be recovered during reprocessing of the

fuel. The low enrichment scheme uses 4 to 5 per cent of uranium-235 and some of the remaining 95 to 96 per cent of uranium-238 is converted during operation to plutonium, which is another recoverable fuel material. The inherent safety of the concrete pressure vessels makes the need for a special containment building, in the view of some designers, unnecessary. The first commercial stations in the USA will however have containment buildings mainly to ease licensing procedures. There is considerable interest in the process heat applications of the HTGR such as coal gasification and steelmaking and in the further development of the HTGR using direct cycle gas turbines instead of the conventional steam cycle.

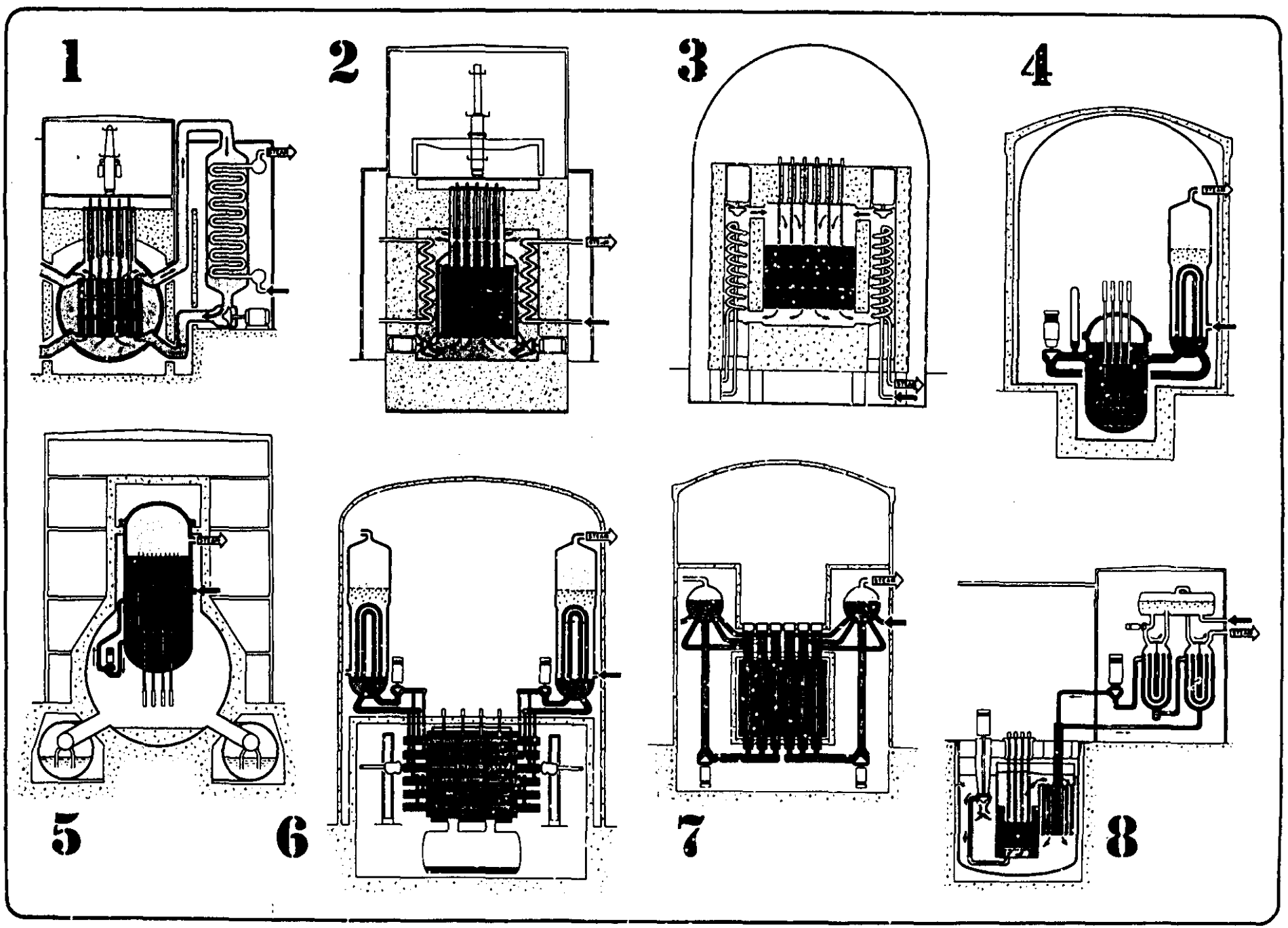
4 PRESSURISED WATER REACTOR. The PWR, now being widely used for nuclear power programmes around the world, was developed initially from the system used to power the US nuclear submarines. Uranium oxide fuel, enriched to between 2 and 4 per cent uranium-235, is contained as sintered pellets in plain tubular cladding. The cladding material in early PWRs was stainless steel but a zirconium alloy is now used. Up to 200 fuel pins 3·5 m in length are mounted on a square lattice to form a single fuel element assembly and a typical reactor core would contain up to 200 such elements. Ordinary water, known as "light water" to distinguish it from heavy water, is used as coolant and at the same time acts as the moderator. To achieve a high coolant outlet temperature without boiling the system must be very highly pressurised. A massive steel pressure vessel with a removable lid is used to contain the relatively compact reactor core. The coolant is circulated through a number of primary loops containing steam generators and pumps. A pressuriser is also connected to one of the loops—it maintains the system pressure by electrical heating or spray cooling of water in a small pressure chamber. The steam generators in most PWRs are of the U-tube type with feedwater recirculation and steam separators and dryers situated in the top of each unit. Once-through straight-tube steam generators are also used in some plants and deliver dry steam direct to the turbine. Control of a PWR is achieved either by neutron absorbing control rods in the form of clusters which can be inserted into each fuel assembly or, for slower power balancing, by varying the concentration of a boron solution in the coolant water. Fuelling of the reactor is carried out at intervals of 12 to 18 months with the reactor shut down and the lid removed from the pressure vessel. The most serious hypothetical accident that could be envisaged with a PWR is a major rupture of a primary loop and a pressure containment building—usually double walled—is adopted to withstand the resulting pressure surge. In some designs an emergency condenser packed with ice is used to reduce the size of the pressure containment building required. Loss of coolant from the reactor system and the resultant depressurisation could result in overheating of the core and emergency core cooling systems are incorporated to deal with such an event.

5 BOILING WATER REACTOR. The BWR is in the same light water reactor family as the PWR and was initially conceived through the consideration of the implications of boiling in the core region of water cooled reactors. It was found that controlled boiling could be achieved in a self-stabilising condition at around half the system pressure of a PWR. The possibility of obtaining steam to drive a turbine direct from the reactor was also demonstrated. BWR systems have now been adopted for commercial nuclear power production in many countries. The reactor, in effect, acts as a recirculation boiler with steam separators and dryers situated in the top section of the reactor pressure vessel. The recirculation flow of water from the steam separators and feedwater returning from the turbine condenser, is usually achieved with jet pumps situated around the reactor core and driven by small external pumps. An alternative design with the impellers of the recirculation pumps situated inside the vessel and sealed drive shafts penetrating the bottom of the vessel, has been introduced in Germany and Sweden. The BWR uses enriched uranium fuel and zirconium alloy cladding with square lattice fuel element assemblies similar to those of the PWR. Control of the reactor is achieved with cruciform rods inserted hydraulically between the fuel assemblies from below the core and also by variation of the recirculation flow rate. The reactor is fuelled off-load at intervals of 12 to 18 months, with the lid of the pressure vessel and steam dryers removed. The core of a BWR is not as compact as a PWR and, together with the jet pumps and steam drying arrangements, this calls for a larger pressure vessel. However, this is offset by the lower operating pressure which allows a thinner walled vessel. A special type of pressure suppression containment is adopted to deal with the consequences of a primary circuit rupture. A pressure relief system in the steel, or concrete, containment chamber around the reactor is arranged to vent steam to a reservoir of water which acts an an emergency condenser. The bulb and torus configuration (shown) is the most familiar arrangement for the pressure suppression system but a simpler arrangement in a cylindrical concrete structure has recently been introduced.

6 CANDU REACTOR. Many heavy water moderated reactor systems have been proposed around the world and several different concepts have been developed to the prototype or demonstration plant size. But the only system now operating on a commercial basis is the Canadian CANDU system. The reactor is fuelled with natural uranium in the form of uranium oxide clad in zirconium alloy. Relatively short fuel element assemblies are made up of 28 fuel pins. The fuel is loaded into horizontal zirconium pressure tubes which pass through a large tank—known as a calandria—filled with heavy water moderator. Heavy water is also used as the coolant but like the PWR it must be maintained at a very high pressure to prevent boiling. The use of pressure tubes for the fuel in the reactor core allows the coolant system to be pressurised without the need for a massive steel pressure vessel. The steam generators are of the U-tube type similar to most PWRs. Like the Magnox reactor, the large natural uranium core requires the use of on-load fuelling and remotely operated machines are used to connect to both ends of the pressure tubes to insert and withdraw the fuel assemblies. Control rods are situated in the moderator penetrating the top of the calandria tank and small power adjustments can also be made by varying the moderator level. An additional method of achieving fast shut down is provided by a dump tank for the heavy water moderator below the calandria. A concrete containment building is used for the Canadian reactors and an unusual additional safety feature is a huge vacuum building designed to suck steam out of the containment building if there is an overpressure.

7 STEAM GENERATING HEAVY WATER REACTOR. The SGHWR system has been developed in Britain with the objective of combining the best features of the CANDU pressure tube reactor and the direct cycle boiling water reactor. A similar system known as CANDU-Boiling Light Water has been developed in Canada and there are also development projects on the same general lines in Italy and Japan. Vertical rather than horizontal pressure tubes are used but otherwise the arrangement of heavy water moderator in a calandria is the same. The difference lies in the use of ordinary light water coolant under controlled boiling conditions in the pressure tubes. Steam is separated from the coolant in steam drums and goes direct to the turbine. Recirculation pumps return the water from the steam drum to the lower end of the pressure tubes. Since light water absorbs more neutrons than heavy water, it is more difficult to maintain a stable nuclear reaction in the core with natural uranium fuel. Most designs, therefore, incorporate a low level of enrichment with uranium-235, or plutonium, in the fuel. The fuel assemblies are very similar to those of a BWR except for the cylindrical dimensions required to fit the pressure tubes. On-load fuelling systems were developed for the 100 MW(e) prototype SGHWR at Winfrith, Dorset, but it is possible to make a case for either on-load or off-load fuelling in a commercial design. A pressure containment building would probably be adopted for any commercial sized reactors of this type.

8 FAST BREEDER REACTOR. The FBR is fuelled with a mixture of plutonium and uranium oxides clad in stainless steel and made up into very compact hexagonal fuel element assemblies. It relies upon the nuclear fission of plutonium and uranium-238 atoms in an intense flux of high energy—or fast—neutrons produced in a highly compact core without any moderator. Having established the fast chain reaction in a compact core of this kind, it is also possible to make use of the very intense neutron radiation emanating from the edge of the core. By placing a blanket of depleted uranium—mostly uranium-238 waste product from enrichment plants—it is possible to convert a substantial quantity of the material to plutonium by neutron capture. It is in fact possible to produce more plutonium in the blanket region than is being consumed in the core region and it is this process which is known as breeding. All countries actively engaged in development of nuclear power programmes—with the exception of Canada—consider that the FBR is the best prospect for generation of electricity in the late 1980s and beyond. The type of FBR at the most advanced stage of development is the Liquid Metal-cooled FBR (LMFBR). The liquid metal used to provide very efficient heat transfer from a compact high power reactor core is sodium. Prototype plants at an advanced stage of construction in Britain and France use the "pool" type configuration shown. The reactor core is suspended in a large pool of sodium together with primary pumps and intermediate heat exchangers. Sodium is pumped from the pool to the bottom of the core and the hot sodium emerging from the top of the core flows down through a straight-tube intermediate heat exchanger and back into the pool. The intermediate heat exchanger heats a secondary flow of sodium which is passed to steam generating plant. The arrangement shown in the simplified diagram has two stage U-tube units for evaporation and superheating of the steam but actual systems under development offer more complex steam cycles including reheat. Other configurations with the primary sodium coolant in external loops are also being adopted for prototype plants. The low pressure of the sodium coolant circuits is considered to be a safety feature once the techniques of handling hot sodium have been mastered. The need to maintain reliable heat removal from the very compact reactor core is the safety problem to which most attention is devoted.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Decay rates

The decay of radioactive isotopes occurs in a random manner. There is a certain probability that in a given time interval a certain fraction of the nuclei of a particular unstable isotope will decay. The rate of decay (dN/dt) is equal to minus the probability of decay, λ, times the number of unstable nuclei present, N.

$$dN/dt = -\lambda N \tag{3.10}$$

The probability of decay is known as the decay constant. Separation of variables allows this simple first-order differential equation to be integrated.

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \tag{3.11}$$

Here N_0 represents the original number of unstable nuclei present at the initial time ($t = 0$) and N is the number of nuclei at some subsequent time, t.

$$ln\left(\frac{N}{N_0}\right) = -\lambda t \tag{3.12}$$

Taking the antilog of each side of Eq. (3.12)

$$N = N_0 e^{-\lambda t} \tag{3.13}$$

It is well to note here that the activity of a sample, A, is the absolute magnitude of its decay rate.

Compound decay

Often the daughter of a radioactive isotope is not stable and decays to a third nuclide, which also may be unstable. Figure 3.1 illustrated the rather lengthy decay chains for Th^{232} and U^{235}.

For the case of an element decaying to an unstable daughter and thence to a third stable isotope, expressions may be written for the number of atoms of each species. If N_{10} represents the original number of parent atoms and N_1 is the number of parent atoms having a decay constant λ_1, at any subsequent time, t,

$$N_1 = N_{10}\, e^{-\lambda_1 t} \tag{3.24}$$

The rate of change of parent nuclei is

$$\frac{dN_1}{dt} = -N_{10}\,\lambda_1 e^{-\lambda_1 t} \tag{3.25}$$

The rate of change of the daughter atoms, dN_2/dt, is due to the buildup caused by the decay of the parent less the decay of the daughter with its own decay constant, λ_2.

$$\begin{aligned}\frac{dN_2}{dt} &= -\frac{dN_1}{dt} - \lambda_2 N_2 \\ &= \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 N_2\end{aligned} \tag{3.26}$$

Rearranging,

$$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_{10} e^{-\lambda_1 t} \tag{3.27}$$

which is an equation of the form

$$\frac{dy}{dx} + a_1(x)y = h(x) \tag{3.28}$$

This first-order ordinary differential equation may be solved through the use of an integrating factor, p.

$$p = e^{\int a_1(x)dx} \tag{3.29}$$

The solution for Eq. (3.28) is

$$y = \left(\frac{1}{p}\right)\int p\, h(x)\, dx + \frac{C}{p} \tag{3.30}$$

where C is a constant of integration.

$$\text{Since } a_1(x) = \lambda_2 \text{ and } p = e^{\int a_1(x)\,dx} = e^{\int \lambda_2 dt} = e^{\lambda_2 t},$$

then

$$N_2 = \left(\frac{1}{e^{\lambda_2 t}}\right) \int e^{\lambda_2 t} \lambda_1 N_{10} e^{-\lambda_1 t} dt + \frac{C}{e^{\lambda_2 t}}$$

$$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}\, e^{-\lambda_1 t} + C\, e^{-\lambda_2 t} \tag{3.31}$$

The constant of integration may be evaluated with the initial conditions, $t = 0$, $N_2 = 0$

$$C = \frac{-\lambda_1 N_{10}}{\lambda_2 - \lambda_1}$$

Thus, the number of daughter atoms may be expressed as

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \tag{3.32}$$

Neutron activation

When a material is placed in a neutron flux it will absorb neutrons in proportion to its cross section. The next heavier isotope will be formed, which may or may not be stable. This results in radioactivity of structural materials in a reactor core, permits the production of sources of radioactivity, and is useful where foils may be inserted at various points in a reactor core to infer the flux level from the resultant activity.

If a newly formed isotope is unstable, it will begin to decay at the same time it is being formed. If N is the number of nuclei of the new isotope, and N_0 is the number of original target nuclei, a differential equation may be written. The rate of change of new nuclei is equal to their rate of formation less their rate of decay.

$$\frac{dN}{dt} = \phi N_0 \sigma_a - \lambda N = \phi \sum_a - \lambda N \tag{8.24}$$

Rearranging,

$$\frac{dN}{dt} + \lambda N = \phi \sum_a$$

This is a first-order differential equation of the form

$$\frac{dy}{dx} + a(x)y = h(x)$$

An integrating factor $p = e^{\int a(x)dx}$ gives a solution of the form

$$y = \left(\frac{1}{p}\right) \int ph(x)\, dx - \frac{C}{p}$$

where C is a constant of integration. In this particular case $a(x) = \lambda$ (constant) and $h(x) = \phi \sum_a$ (constant).

$$p = e^{\int \lambda dt} = e^{\lambda t}$$

$$N = \frac{1}{e^{\lambda t}} \int e^{\lambda t} \phi \sum_a dt - \frac{C}{e^{\lambda t}}$$

$$= \frac{\phi \sum_a}{\lambda} - \frac{C}{e^{\lambda t}}$$

$$C = \frac{e^{\lambda t} \phi \sum_a}{\lambda} + e^{\lambda t} N$$

Starting with an unirradiated sample where when $t = 0$ and $N = 0$, the constant of integration may be evaluated:

$$C = \frac{e^0 \phi \sum_a}{\lambda} - e^0 N = \frac{\phi \sum_a}{\lambda}$$

$$N = \frac{\phi \sum_a}{\lambda}(1 - e^{-\lambda t}) \tag{8.25}$$

Figure 8.5 shows the buildup of radioactive nuclei during irradiation in a neutron flux. When the irradiated sample is removed from the core, it will decay with its characteristic half-life.

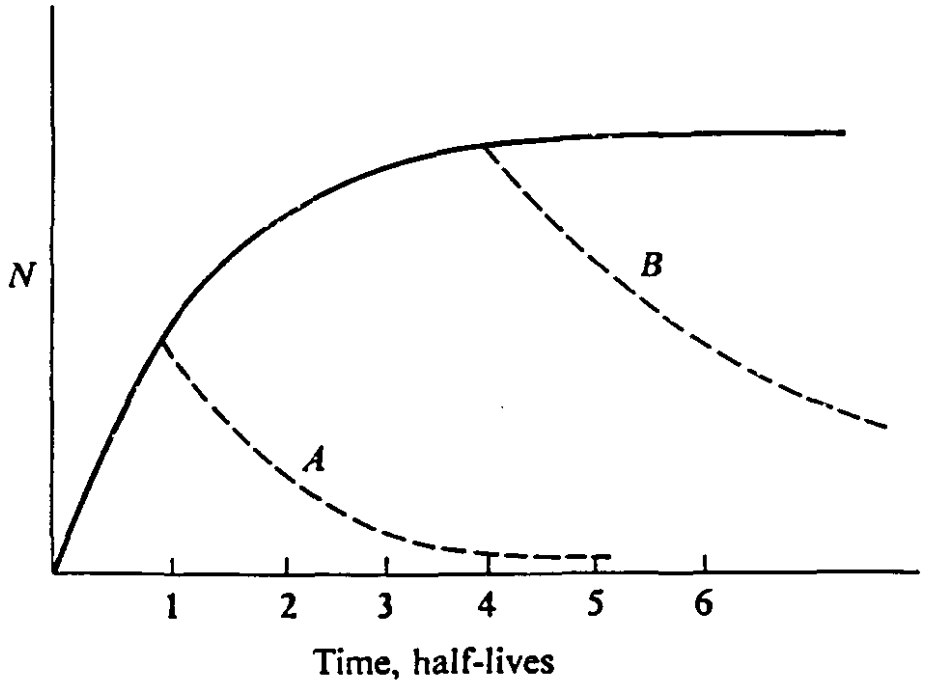

FIG. 8.5 Buildup of radioactive nuclei during irradiation. Curves A and B represent decay upon removal after 1 and 4 half-lives of irradiation.

Attenuation of a neutron beam

Consider a collimated beam of neutrons impinging perpendicularly on a surface of area A, as shown in Fig. 8.1.

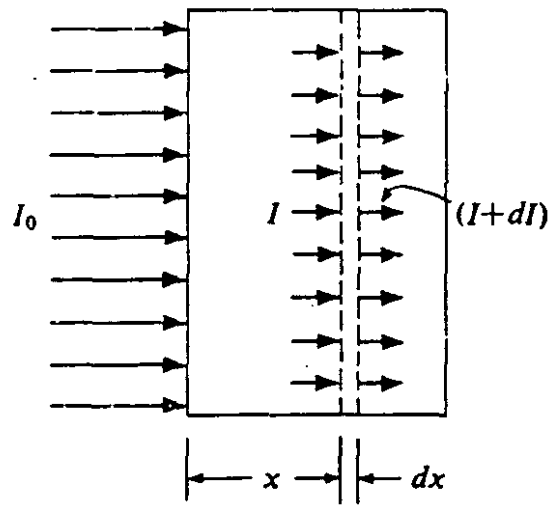

FIG. 8.1 Collimated beam of neutrons impinging on material with cross sectional area of A, cm². Note that, since neutrons are being removed, dI will be negative.

A = surface area, cm²
x = distance from front face, cm
I_0 = initial beam intensity, n/cm² sec
I = intensity at distance x from front face, n/cm² sec
σ = microscopic cross section, cm²/nucleus
N = density of target atoms, nuclei/cm³
$NA\,dx$ = number of target atoms in the differential thickness, dx.
$\sigma NA\,dx$ = total effective area presented by nuclei to the neutrons, cm².

The ratio of the total effective area in the differential slab to the full area gives the probability of interaction in the distance dx.

$\sigma NA\,dx/A$ = probability of interaction (fraction of beam that undergoes an interaction), cm² effective area/cm² total area

The decrease in intensity, dI, as the neutrons pass through the differential slab, is the intensity at that point times the probability of interaction.

$$dI = -I\sigma N\,dx \tag{8.1}$$

Separating variables,

$$\frac{dI}{I} = -\sigma N\,dx$$

Then, integrating for a thickness x

$$\int_{I_0}^{I} \frac{dI}{I} = -\sigma N \int_0^x dx$$

$$\ln\left(\frac{I}{I_0}\right) = -\sigma N x$$

$$I = I_0 e^{-\sigma N x} = I_0 e^{-\Sigma x} \tag{8.2}$$

Where $\Sigma = \sigma N$ = macroscopic cross section, cm^2/cm^3. This represents the effective target area per unit volume of material.

The number density of the target atoms may be found by the product of the density multiplied by Avagadro's number divided by the mass number.

$$N = \frac{\rho \text{ gm/cm}^3 \times 6.023 \times 10^{23} \text{ atoms/gram atom}}{A \text{ gm/gram atom}}$$

$$N = \frac{6.023 \times 10^{23}\rho}{A} \tag{8.3}$$

Differentiating Eq. (8.2),

$$\frac{dI}{dx} = -I_0 \Sigma e^{-\Sigma x} \tag{8.4}$$

Thus, the rate of absorption decreases exponentially.

Mean free path

The average distance travelled by a neutron before interaction is known as the mean free path, λ. From Eq. (8.4) it may be seen that the decrease in intensity while travelling a distance dx is due to the number of neutrons interacting and, hence, removed from the beam.

$$dI = -I_0 \Sigma e^{-\Sigma x}\,dx$$

These neutrons have travelled a distance x without interaction. The total distance travelled by all the neutrons as they interact in an infinite thickness of material is

$$-\int_{x=0}^{x=\infty} x\,dI = +I_0 \Sigma \int_0^\infty x e^{-\Sigma x}\,dx$$

This represents the summation for the infinite slab of the distance travelled by neutrons absorbed in each differential thickness. The mean free path is then this total interaction distance divided by the original beam intensity.

$$\lambda = \frac{+I_0 \Sigma \int_0^\infty x e^{-\Sigma x}\,dx}{I_0} = \Sigma \int_0^\infty x e^{-\Sigma x}\,dx = \frac{1}{\Sigma} \tag{8.5}$$

Thus, the reciprocal of the macroscopic cross section (cm^2/cm^3) is the mean free path (cm). This same result is useful for considering neutron flux where the neutrons are not collimated, but are travelling in random directions.

Since cross sections are probabilities of interaction, individual probabilities may be summed to give a total probability. The total cross section is the sum of the absorption and scatter cross sections

$$\sigma_T = \sigma_a + \sigma_s \tag{8.6}$$

In turn, for a fissionable nucleus the absorption cross section is the sum of the fission and radiative capture cross sections.

$$\sigma_a = \sigma_f + \sigma_c \tag{8.7a}$$

Similarly, the scattering cross section is made up of an elastic plus an inelastic value.

$$\sigma_s = \sigma_{se} + \sigma_{si} \tag{8.7b}$$

Macroscopic cross sections may also be added:

$$\Sigma_T = \Sigma_f + \Sigma_c + \Sigma_s = \frac{1}{\lambda_f} + \frac{1}{\lambda_c} + \frac{1}{\lambda_s} = \frac{1}{\lambda_T} \tag{8.8a}$$

Thus, the total mean free path is

$$\lambda_T = \frac{\lambda_f \lambda_s \lambda_c}{\lambda_c \lambda_s + \lambda_f \lambda_s + \lambda_f \lambda_c} \tag{8.8b}$$

The relaxation length for a material is the thickness of material necessary to attenuate the neutron beam by a factor of e. Setting $x = \lambda$

$$I_\lambda = I_0 e^{-\Sigma\lambda} = I_0 e^{-1} = \frac{I_0}{e} \tag{8.9}$$

Therefore, the mean free path is also the relaxation length.

Development of diffusion equation

As neutrons diffuse through a reactor core they may (i) be absorbed by fuel, moderator, coolant, cladding, structure, etc.; (ii) leak out at the core boundaries; or (iii) act as a source for new fission neutrons. If one considers the neutrons in a differential volume, dV, an expression can be

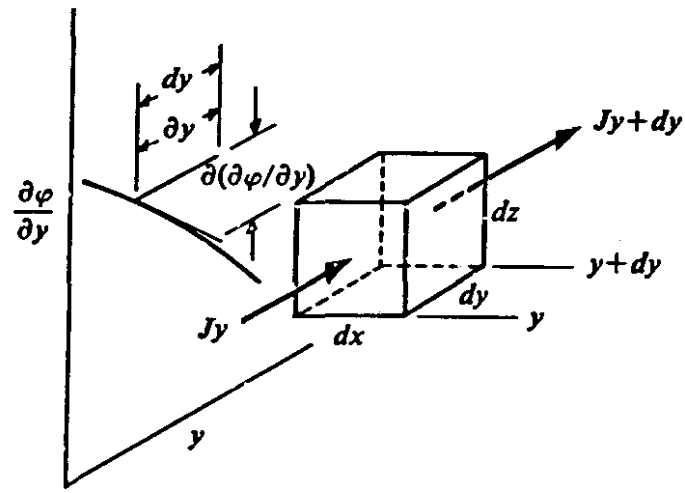

FIG. 9.6 Neutron leakage in the y-direction from a differential volume.

developed for the net neutron leakage from this elemental volume. First consider only neutrons leaking into the front face in the y direction.

$$L_y = J_y\,dx\,dz = -D\left(\frac{\partial\phi}{\partial y}\right)dx\,dz \tag{9.34}$$

From the rear face the leakage is

$$L_{y+dy} = J_{(y+dy)}\,dx\,dz = -D\left(\frac{\partial\phi}{\partial y} + \frac{\partial(\partial\phi/\partial y)}{\partial y}dy\right)dx\,dz \tag{9.35}$$

The net leakage in the y direction is the difference between (9.35) and (9.34).

$$L_{y_{net}} = L_{y+dy} - L_y = -D\left[\frac{\partial\phi}{\partial y} + \frac{\partial^2\phi}{\partial y^2}dy\right]dx\,dz + D\frac{\partial\phi}{\partial y}dx\,dz = -D\frac{\partial^2\phi}{\partial y^2}dx\,dy\,dz \tag{9.36}$$

Similarly,

$$L_{x_{net}} = -D\frac{\partial^2\phi}{\partial x^2}dx\,dy\,dz \tag{9.37}$$

$$L_{z_{net}} = -D\frac{\partial^2\phi}{\partial z^2}dx\,dy\,dz \tag{9.38}$$

The total leakage for a unit volume (neutrons/cm³ sec) is

$$L_t = -D\left[\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}\right] = -D\nabla^2\phi \tag{9.39}$$

Neutrons as Waves

Earlier in this book, in Chapter 11, we discussed the wave nature of the electron and mentioned that the argument could equally well be applied to any free particle, the de Broglie wavelength being given by

$$\lambda = \frac{h}{mv}.$$

This is true for any particle having momentum equal to mv.

For electrons, $m = 9{\cdot}1 \times 10^{-31}$ kg
$e = 1{\cdot}6 \times 10^{-19}$ coulomb
and $h = 6{\cdot}6 \times 10^{-34}$ joule s

When an electron is accelerated through V volts the energy equation is $Ve = \frac{1}{2}mv^2$ (if V is of the order of a few kilovolts only), so that

$mv = \{2Vem\}^{\frac{1}{2}}$ and the wavelength then becomes, in metres,

$$\lambda = \frac{h}{(2Vem)^{\frac{1}{2}}} = \frac{6{\cdot}6 \times 10^{-34}}{(2 \times V \times 1{\cdot}6 \times 10^{-19} \times 9{\cdot}1 \times 10^{-31})^{\frac{1}{2}}}$$

$$= \frac{6{\cdot}6}{(29 \times V)^{\frac{1}{2}}} \times 10^{-9} \text{ m}$$

$$= \sqrt{\frac{1{\cdot}5}{V}} \text{ nm which is a convenient expression for } \lambda \text{ with } V \text{ in}$$

volts, for electrons only.

In the case of a neutron beam

$$\lambda = \frac{h}{mv} \text{ becomes } \frac{h}{\sqrt{2mE}}$$

$$= \frac{6{\cdot}6 \times 10^{-34}}{(2 \times E \times 1{\cdot}6 \times 10^{-19} \times 1{\cdot}66 \times 10^{-27})^{\frac{1}{2}}} \text{ metres}$$

where E is converted to electron volts, giving $\lambda = \dfrac{28{\cdot}6}{\sqrt{E}}$ pm, for neutrons.